Web-appendix for:

Identifying Modern Macro Equations with Old Shocks

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1 Derivation of Exogeneity and Relevance conditions

The stationarity and uncorrelated assumptions imply that

$$\mathrm{E}(\xi_t^i \xi_s^i) = \begin{cases} \sigma_\xi^2 & t = s \\ 0 & t \neq s \end{cases},$$

for all $t, s = 1, \ldots, n$. Additionally, from the linearity assumption we have that we can write for each endogenous variable $w_{j,t} = \beta^{j'} \xi_{t:t-H}^i + \eta_t^j$, for $j = 1, 2, 3$. Similarly, for the error term $u_t = \beta^{u'} \xi_{t:t-H}^i + \eta_t^u$. The disturbances η_t^j and η_t^u are mean zero and uncorrelated with $\xi_{t:t-H}^i$.

Next, we rewrite the exogeneity assumption. We have that for each $h = 0, \ldots, H$

$$\mathrm{E}(\xi_{t-h}^i u_t) = \mathrm{E}(\xi_{t-h}^i (\beta^{u'} \xi_{t:t-H}^i + \eta_t^u)) = \sigma_\xi^2 \beta_h^u \ .$$

Since, $\sigma_\xi^2 > 0$, the exogeneity condition can only be satisfied when $\beta_h^u = 0$ for all $h = 0, \ldots, H$.

For the relevance condition we have for $j = 1, 2, 3$ and $h = 0, \ldots, H$ that

$$\mathrm{E}(\xi_{t-h}^i w_{j,t}) = \mathrm{E}(\xi_{t-h}^i(\beta^{j'}\xi_{t:t-H}^i + \eta_t^j)) = \sigma_\xi^2 \beta_h^j \ .$$

Using this we obtain

$$\mathrm{E}(\xi_{t:t-H}^i w_t') = \sigma_\xi^2 \begin{bmatrix} \beta_0^1 & \beta_0^3 \\ & \\ \beta_H^1 & \beta_H^3 \end{bmatrix}$$

and it follows that requiring $\mathrm{E}(\xi_{t:t-H}^i w_t')$ to be full column rank is equivalent to requiring $[\beta_h^1, \beta_h^2, \beta_h^3]_{h=0}^H$ to be full column rank (or linearly independent).

2 Additional empirical results

As a preliminary, figure 1 reports the impulse responses of unemployment and PCE inflation estimated from a distributed lags model with the Romer and Romer monetary shocks over 1969–2007. Note how the response of inflation is stronger than the response of unemployment; this explains why (in figure 1 of the main text) the coefficient γ_f is more precisely estimated than the coefficient λ.

We now report additional empirical results on the estimation of the New-Keynesian Phillips curve.

First, we consider using core inflation instead of headline inflation. A popular usual rational for using core inflation —inflation excluded food and energy prices— instead of headline inflation in estimates of the Phillips curve is to improve estimation precision. Because food and energy prices can display large idiosyncratic transitory movements unrelated to the business cycle, the NKPC estimates based on headline inflation may be less precise than NKPC estimates based on core inflation.

Figure 2 shows the confidence sets for λ and γ obtained using core PCE inflation and

the Romer-Romer monetary shock instrument for 1969-2007. The results are similar to our results with headline inflation with 2SLS point estimates at $\lambda \approx -.64$ and $\gamma_f \approx .36$.

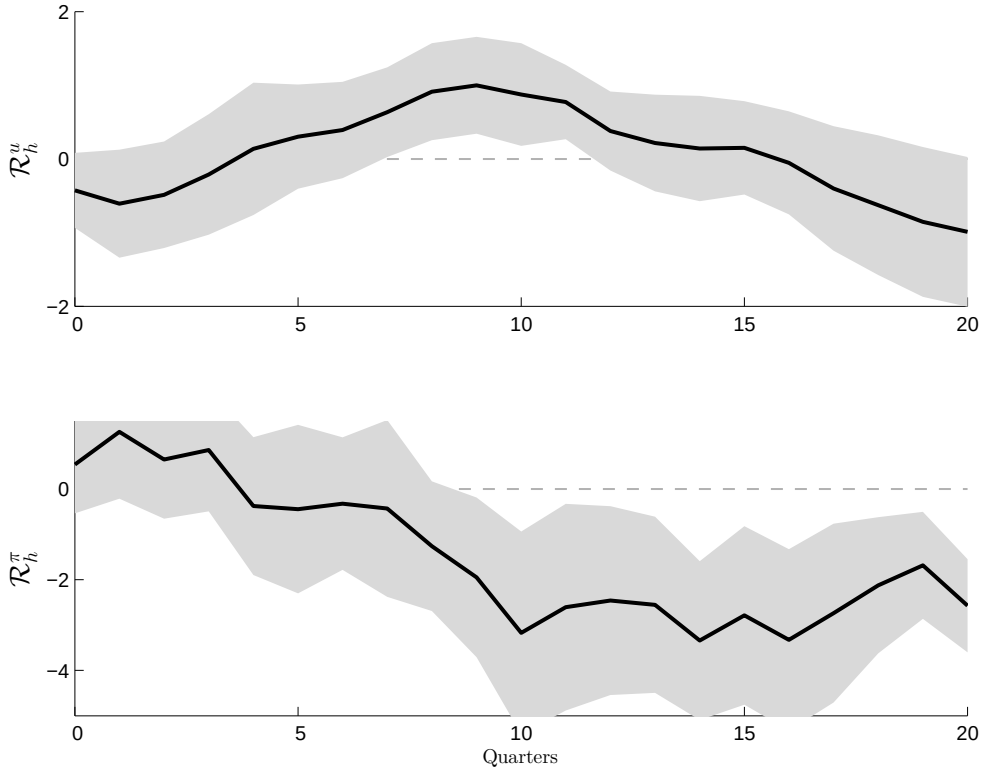
Looking at the more recent (and thus arguably more relevant for current policy making) sample, we also consider the effect of using core PCE inflation instead of headline inflation on the NKPC coefficients. As shown in Figure 3, the estimates are slightly more precise but the confidence sets point in the exact same direction as with headline inflation, and the conclusions are identical: the slope of the NKPC is no longer significant after 1990, but the coefficient on expected future inflation becomes significant.

While the main text focused on the HFI instrument post-1990, we can also estimate the NKPC using the RR instruments post-1990. Figure 4 shows the results and gives again the same conclusion: the slope of the NKPC is no longer significant after 1990, but the coefficient on expected future inflation is substantially larger post-1990.

We then consider using the output gap instead of unemployment as the forcing variable. We define the output gap as the log deviation of real GDP from its natural level, as estimated from an HP filter with $\lambda^{hp} = 1600$. Figure 5 and 6 plot the estimated confidence using core inflation and respectively the HFI and the RR monetary shocks over the post-1990 smaple period. The results are remarkably similar to what we obtained with the unemployment rate. And as with the unemployment rate, the slope of the Phillips curve is substantially underestimated by OLS (in absolute value), while the coefficient on expected future inflation is substantially overestimated by OLS.

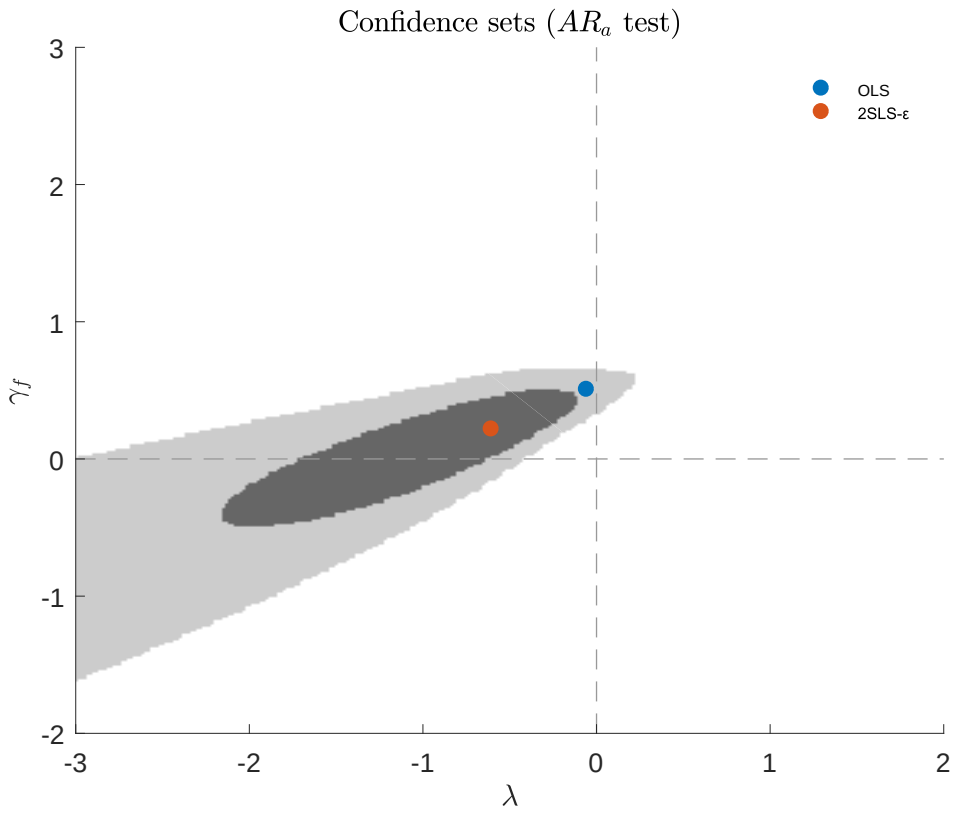
Finally, figures 7 and 8 plot the estimated Phillips curve using the output gap as the forcing variable and treating lagged inflation as endogenous. The results are broadly similar to the ones obtained using unemployment, although uncertainty about γ_b is larger.

Figure 1: IRs of inflation and unemployment to a monetary shock



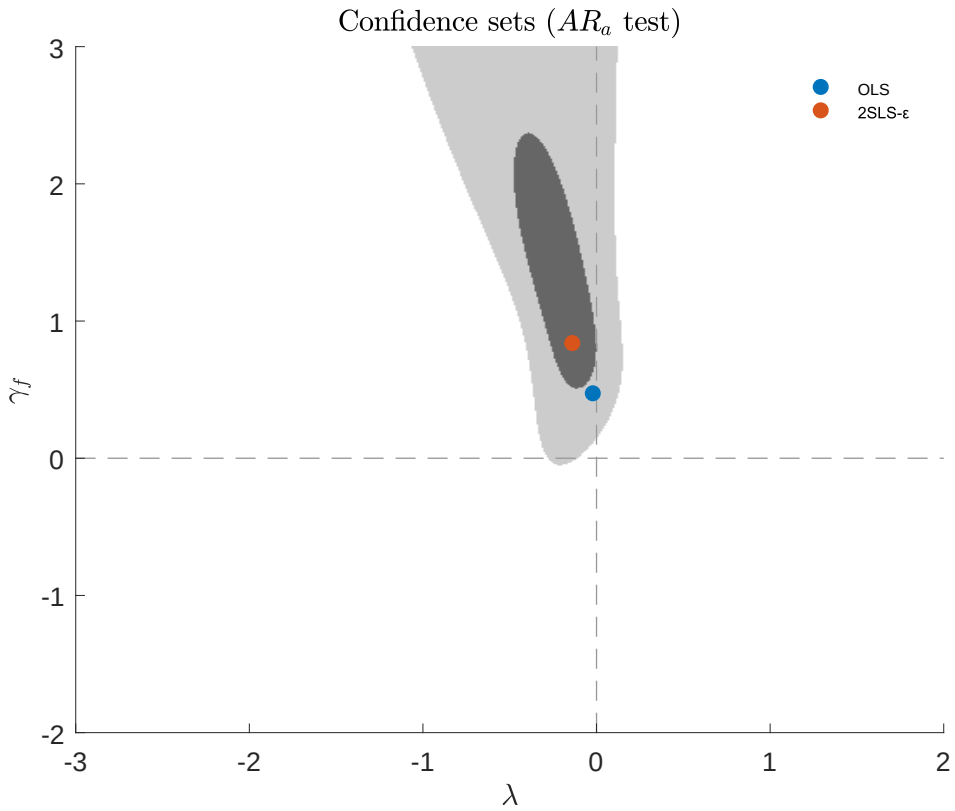
Notes: Impulse responses for u_t and π_t computed using the Romer and Romer narrative monetary shocks over 1969q1–2007q4.

Figure 2: The Phillips curve — RR id. (1969-2007), core PCE



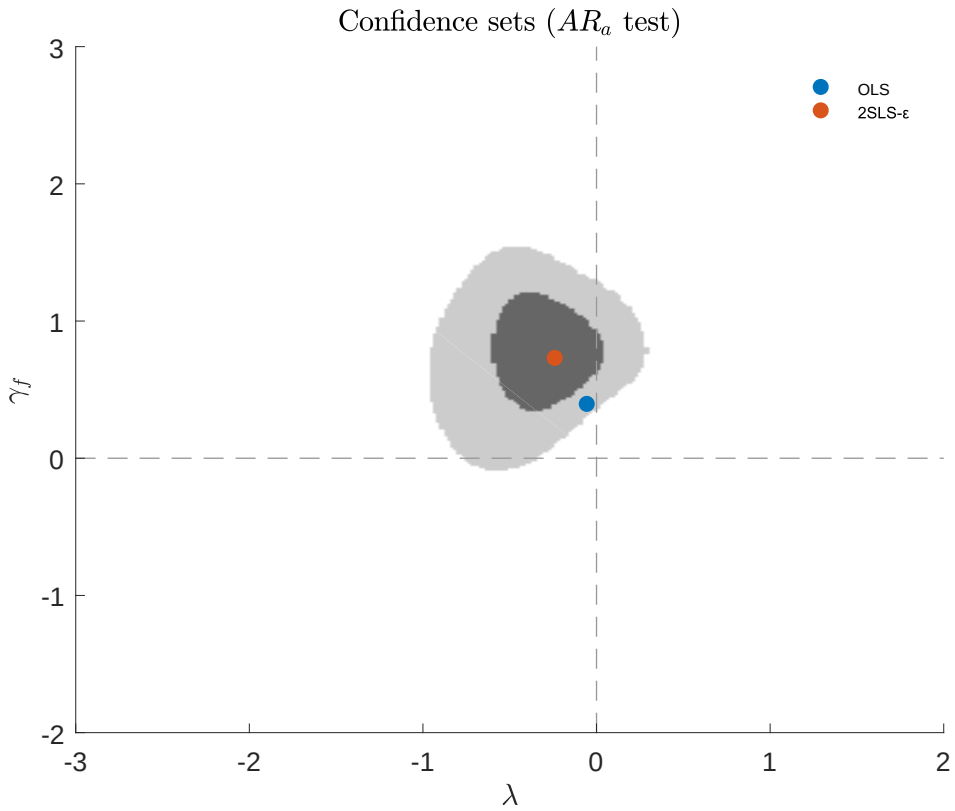
Notes: 95 and 68 percent robust confidence sets for the Phillips curve (using core PCE inflation) coefficients obtained by inverting the AR_a tests over the parameter space of λ and γ_f. Estimation based on using the Romer-Romer (RR) monetary shocks as instruments over 1969-2007. The red dot (2SLS-ε) is the Almon-restricted 2SLS estimate using lags of the Romer and Romer monetary shocks as instruments, the green dot (2SLS-GIV) is the 2SLS estimate using 2 lags of inflation and unemployment as instruments, the blue dot is the OLS estimate.

Figure 3: The Phillips curve — HFI id. (1990-2007), core PCE



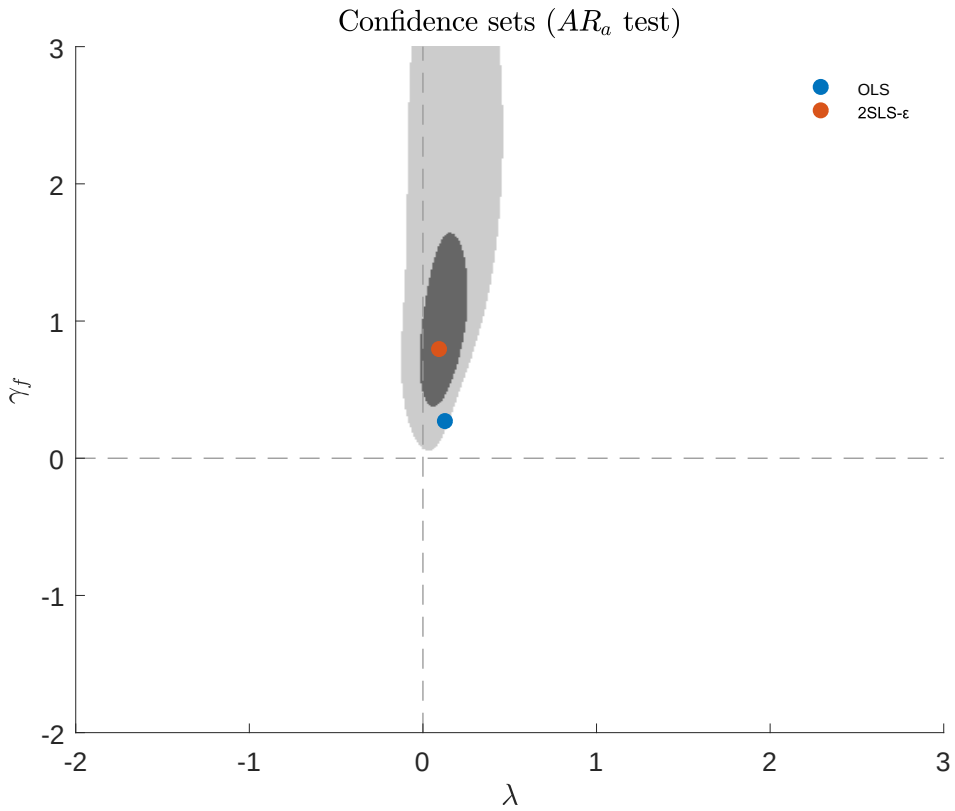
Notes: 95 and 68 percent robust confidence sets for the Phillips curve coefficients obtained by inverting the AR_a tests over the parameter space of λ and γ_f. Estimation based on using the HFI monetary surprises as instruments over 1990-2007. The red dot (2SLS-ε) is the Almon-restricted 2SLS estimate using lags of the HFI monetary shocks as instruments.

Figure 4: The Phillips curve — RR id. (1990-2007), core PCE



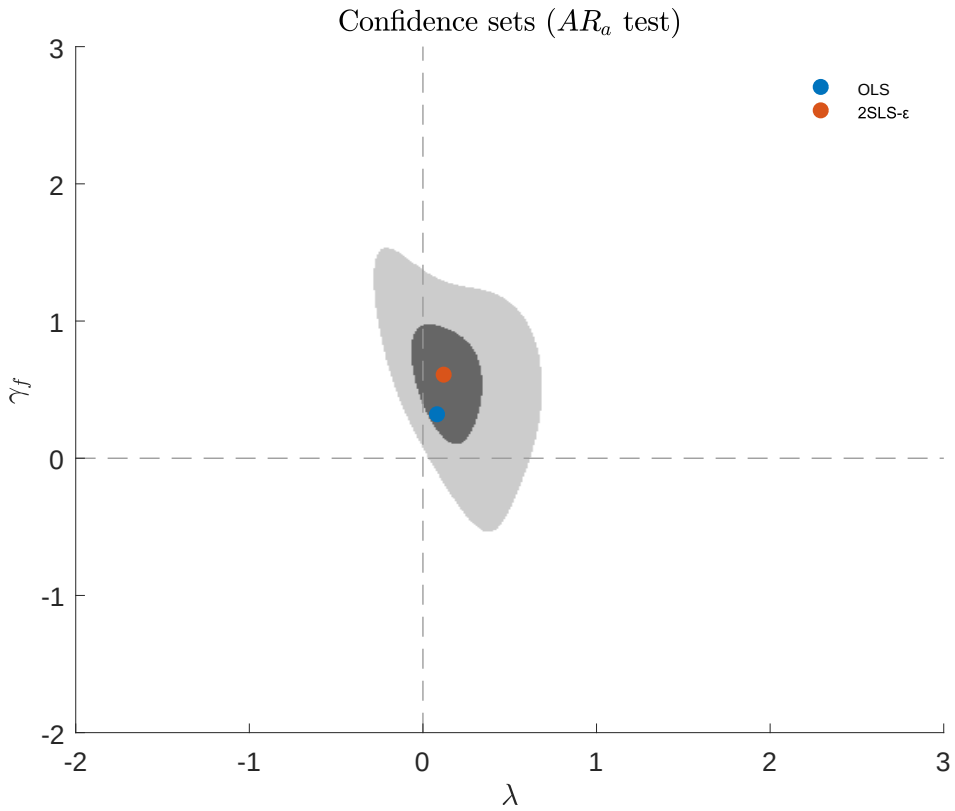
Notes: 95 and 68 percent robust confidence sets for the Phillips curve coefficients obtained by inverting the AR_a tests over the parameter space of λ and γ_f. Estimation based on using the HFI monetary surprises as instruments over 1990-2007. The red dot (2SLS-ε) is the Almon-restricted 2SLS estimate using lags of the Romer and Romer monetary shocks as instruments.

Figure 5: The output gap Phillips curve — HFI id. (1990-2007), core PCE



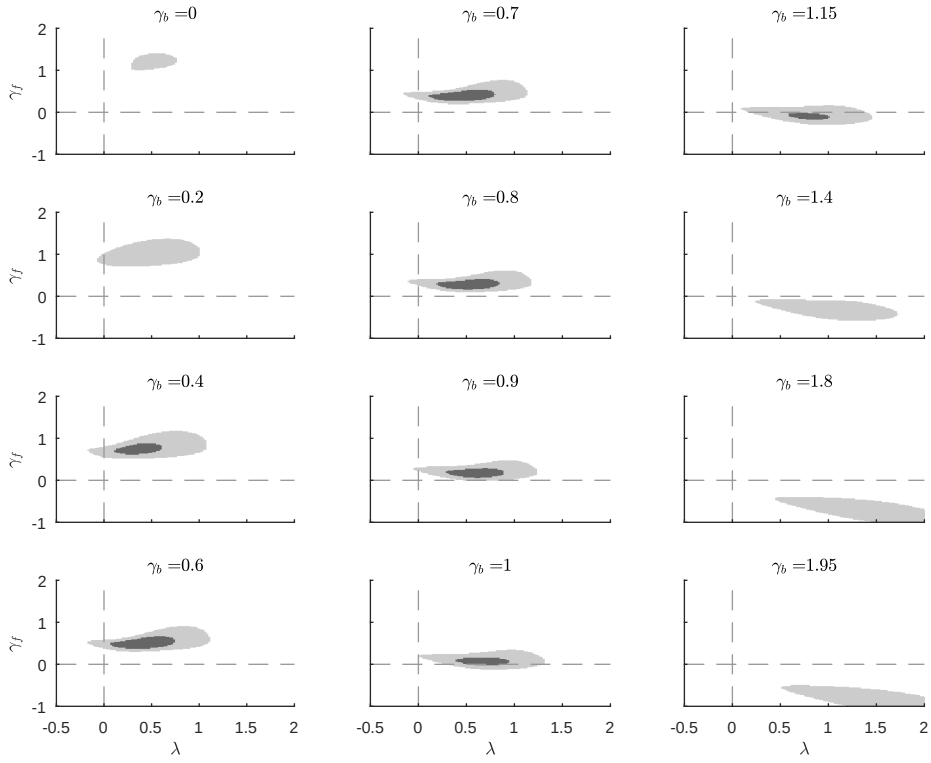
Notes: 95 and 68 percent robust confidence sets for the Phillips curve coefficients obtained by inverting the AR_a tests over the parameter space of λ and γ_f. Estimation based on using the HFI monetary surprises as instruments over 1990-2007. The red dot (2SLS-ε) is the Almon-restricted 2SLS estimate using lags of the HFI monetary shocks as instruments.

Figure 6: The output gap Phillips curve — RR id. (1990-2007), core PCE



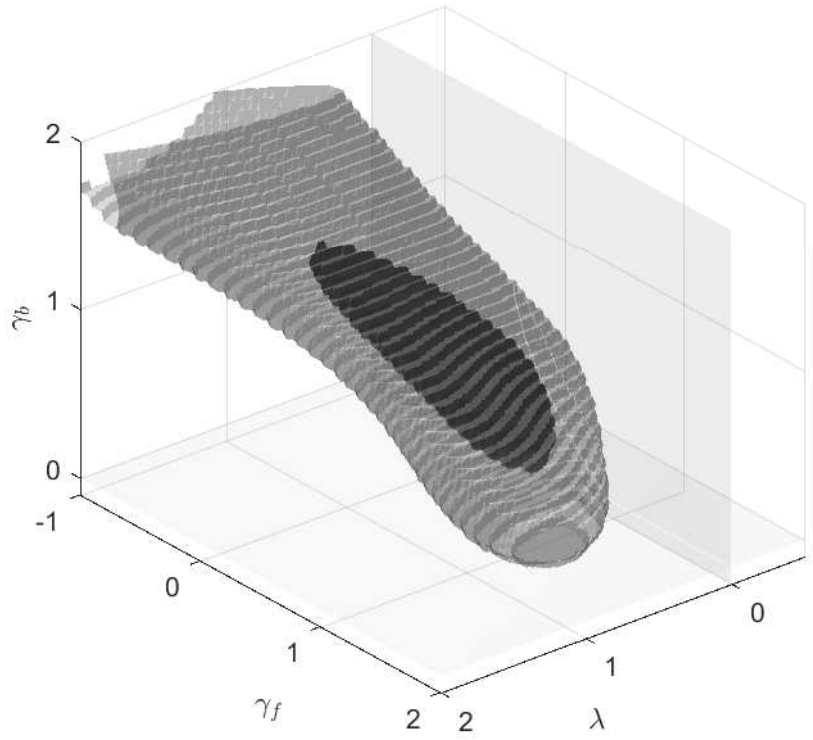
Notes: 95 and 68 percent robust confidence sets for the Phillips curve coefficients obtained by inverting the AR_a tests over the parameter space of λ and γ_f. Estimation based on using the HFI monetary surprises as instruments over 1990-2007. The red dot (2SLS-ε) is the Almon-restricted 2SLS estimate using lags of the Romer and Romer monetary shocks as instruments.

Figure 7: The output gap Phillips curve — 1969-2007, RR id., π_{t-1} endogenous



Notes: Robust confidence sets for the Phillips curve coefficients on unemployment (λ), expected future inflation (γ_f) and past inflation (γ_b). Estimation based on using the Romer-Romer (RR) monetary shocks as instruments over 1969-2007. 68 percent (dark grey) and 95 percent (light grey) confidence sets obtained by inverting the AR_s tests over the parameter space of λ, γ^f and γ^b.

Figure 8: The output gap Phillips curve — 1969-2007, RR id., π_{t-1} endogenous



Notes: Robust confidence sets for the Phillips curve coefficients on unemployment (λ), expected future inflation (γ_f) and past inflation (γ_b). Estimation based on using the Romer-Romer (RR) monetary shocks as instruments over 1969-2007. 68 percent (dark grey) and 95 percent (light grey) confidence sets obtained by inverting the AR_s tests over the parameter space of λ, γ^f and γ^b.